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GENERATION OF A PLANE RELAXATION WAVE IN AN AEROCOLLOIDAL SUSPENSION OF SOLID PARTICLES

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The analysis of propagation of a stationary shock wave in an aerocolloidal suspension [1-4] has shown that behind the shock front is a rather broad relaxation zone, in which the suspended particles are gradually accelerated by the gas flow. In that zone the particles are heated up to the temperature of the gas, heat is released due to the work of friction forces, and various phase transitions are possible, for example, melting and evaporation of the colloidal particles. It is exceedingly difficult to obtain an analytic solution of the system of differential equations describing the gas and particles; as a rule, a computer is recruited as an aid to finding solutions for various special cases.

Even more insurmountable are the mathematical difficulties associated with investigation of the transient part of shock generation in an aerocolloid, as in the case, for example, when a shock wave traveling through a pure gas impinges on a domain filled with an aerocolloid.

For a small volume concentration of particles the leading edge of the shock wave enters the aerocolloid virtually unchanged. Immediately, however, two contact surfaces are formed: 1) the boundary of the moving cloud of particles; 2) the boundary (interface) between the original (before arrival of the shock) dusty gas and the clean gas.* The particles set in motion generate disturbances in the surrounding medium in the form of rarefaction and compression waves. Inasmuch as the leading edge of the shock waves moves relative to the trailing gas at less than the velocity of sound, the disturbances overtake the shock front and begin to deform it. Finally a reflected shock is formed and propagates in the opposite direction.

*The second boundary is logically called the gas contact surface. Its trajectory is clearly the trajectory of the gas mass present at the initial instant at the nonmoving interface between the gas and aerocolloid.

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It must also be recognized that the aerocolloidal particles play a dual role from the thermodynamic point of view. On the one hand, they function as heat sinks when warmed by the compressed and shock-heated gas. On the other hand, the dissipation of energy through friction between the gas and the particles causes the latter to release heat from their surface and to act as an additional heat source for the gas. This dual role of the particles in the total heat balance can produce a nonmonotonic temperature variation in the relaxation zone.

Various methods can be used with success to determine the characteristic attributes of the relaxation wave generated in an aerocolloid. However, before running through countless computational variants with different initial parameters in order to exhibit definite quantitative laws it would be desirable to determine by approximative methods a qualitative pattern of the influence of all such parameters on the resulting shock structure. This approach to the solution of the stated problem is described below.

§1. Statement of the Problem. The method developed here is based on an iterative procedure with a special scheme for separation of the variables characterizing the state and motion of the gas from the state and motion variables of the solid phase.

To facilitate the determination of all the qualitative laws we first introduce some simplifications. We consider the gas to be ideal, obeying the Mendeleev–Clapeyron equation and having a definite adiabatic exponent $\gamma = c_p/c_v$. We treat the set of particles as a continuum. For a small volume concentration $\rho_p/\rho_P < 10^{-3}$, where ρ_p is the mass density of the dispersed component and ρ_P is the density of the particle material, this set can be regarded as a perfectly plastic inviscid gas. The particles are small enough to prevent the colloid from settling appreciably during the transit time of the relaxation wave. All particles have the same radius. The thermal conductivity of the solid component is much greater than the thermal conductivity of the gas, so that the temperature gradients inside a particle are negligible. The local variations of the velocity field of the gas are concentrated solely within the immediate proximity of the given particle and do not affect its neighbors. The relaxation times for the velocity and particle heating are determined by the viscosity and thermal conductivity of the gas* and are equal to, respectively,

$$\tau_u = \frac{1}{18}\frac{\rho_P d^2}{\mu}, \qquad \tau_T = \frac{1}{12}\frac{\rho_P c_P d^2}{k},$$

where ρ_P and c_P are the density and specific heat of the particle material; d is the particle diameter; and μ, k are the viscosity and thermal conductivity of the gas; thus, the two times are close to one another in practice.

The restrictions imposed on the diameter d and mass density ρ_p by the conditions of continuity of the dispersed component, isolation of the particles, and absence of gravity settling of the particles can be expressed as inequalities for the specific system of sand + air under standard conditions:

$$10^{-18} d^{-3} \text{ kg/m}^3 < \rho_p < 10 \text{ kg/m}^3, \quad d < 10^{-3} \text{ m}.$$

Under the given assumptions the aerocolloid can be treated as a set of two interpermeating gases, and the mass, momentum, and energy balance equations for each one written separately in the form [3]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\rho u = 0; \tag{1.1}$$

$$\frac{\partial}{\partial t}\rho u + \frac{\partial}{\partial x}\left(\rho u^2 + \frac{p}{\gamma}\right) = \rho_p (u_p - u) f \mu^0(T); \tag{1.2}$$

$$\frac{\partial}{\partial t}\rho\left[T + \gamma(\gamma-1)\frac{u^2}{2}\right] + \frac{\partial}{\partial x}\rho u\left[T + \gamma(\gamma-1)\frac{u^2}{2} + (\gamma-1)\frac{p}{\rho}\right] = \rho_p[\gamma(\gamma-1)u_p(u_p-u)f + \lambda c(T_p - T)g]\mu^0(T); \tag{1.3}$$

$$p = \rho T; \tag{1.4}$$

$$\frac{\partial}{\partial t}\rho_p + \frac{\partial}{\partial x}\rho_p u_p = 0; \tag{1.5}$$

$$\frac{\partial}{\partial t}\rho_p u_p + \frac{\partial}{\partial x}\rho_p u_p^2 = -\rho_p(u_p - u) f\mu^0(T); \tag{1.6}$$

$$\frac{\partial}{\partial t}\rho_p\left[cT_p + \gamma(\gamma-1)\frac{u_p^2}{2}\right] + \frac{\partial}{\partial x}\rho_p u_p\left[cT_p + \gamma(\gamma-1)\frac{u_p^2}{2}\right] = -\rho_p[\gamma(\gamma-1)u_p(u_p-u)f + \lambda c(T_p - T)g]\mu^0(T), \tag{1.7}$$

where ρ, u, p, and T are the density, velocity, pressure, and temperature of the true gaseous component; the subscript p refers to the parameters of the dispersed component; $\lambda = \tau_u/\tau_T = (2/3)\gamma/c \text{ Pr}$ is the ratio of the characteristic relaxation times; $c = c_P/c_v$ is the relative specific heat of the particle material; Pr is the Prandtl

*If necessary, additional allowance can be made for the influence of the Reynolds and Mach numbers on the given relaxation times.

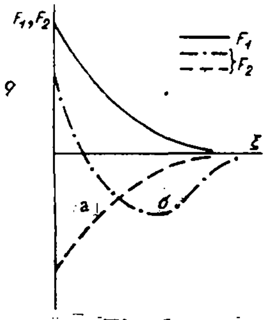

Fig. 1

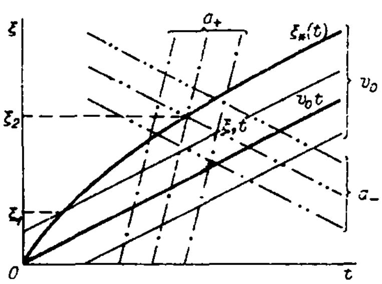

Fig. 2

number; $\mu^0(T) = \mu(T)/\mu(T_0)$ is the relative dynamic viscosity of the gas; and f, g are quantities characterizing the departure of the particle drag from the Stokes law and of the heat transfer from the regime Nu = 2. We assume [5] that $f = (1 + Re^{1/2}/6 + Re/60)[1 + \exp(-0.427/M^{4.63} - 3.0/Re^{0.88})]$, $g = 1 + 0.3 Pr^{1/3} Re^{1/2}$. These equations are given in dimensionless form with the values of ρ, p, T and the speed of sound a in the undisturbed gas taken as the scales of the dependent variables, and with the characteristic time τ_u and relaxation length $\lambda_u = a\tau_u$, also calculated from the rest-gas parameters, taken as the natural units of time and space measurement.

It is seen that the complete system of differential equations (1.1)-(1.7) decomposes into two subsystems, one describing the gaseous component and the other the dispersed component. The only coupling between the subsystems is through the right-hand sides of the equations, i.e., the volume sources. This fact enables us to formulate the following iterative scheme. Assuming first that the particles do not influence the gas flow, we use (1.5)-(1.7) to determine the distributions of the parameters of the particle cloud behind the shock wave. Using the description thus obtained for the flow of the dispersed component, we then find the perturbations of the gas parameters from the subsystem (1.1)-(1.4) and try once again to solve (1.5)-(1.7) so as to refine the previously obtained distribution of the parameters of the particles in the shock wave, and so on. To determine the qualitative pattern of events taking place in the generation of the relaxation wave it is sufficient to analyze the results obtained after the first iteration step.

§2. Analytical Solution. Let a plane shock wave with an infinite front and specified parameters p_0, u_0, ρ_0, T_0 impinge at time t = 0 on a cloud of particles occupying the half-space x > 0 and existing in equilibrium with a gas at rest. We adopt a coordinate system $\xi = w_0 t - x$ moving together with the shock front and we introduce velocities $v_p = w_0 - u_p$ and $v = w_0 - u$. Since we are assuming that the particles do not initially affect the gas component in the domain occupied by the particles, the quantities ρ_p, v_p, and T_p do not depend on the time, and so the subsystem (1.5)-(1.7) is readily integrated. We assume, accordingly, that the profiles of ρ_p, v_p, and T_p behind the shock front are known, as are the laws governing the motion of the gas-aerocolloid contact surface $\xi_*(t)$.

We now consider the influence of the moving dispersed component on the parameters of the gas in the shock wave. If we assume a small particle concentration, then the disturbances introduced into the gas flow are small. In this case, setting

$$\rho_1 = (\rho - \rho_0)/\rho_0,\ T_1 = (T - T_0)/T_0,\ p_1 = (p - p_0)/p_0,\ v_1 = (v - v_0)/a_0,$$

we write the subsystem (1.1)-(1.4) in the linearized form

$$\frac{\partial \rho_1}{\partial t} + v_0 \frac{\partial \rho_1}{\partial \xi} + a_0 \frac{\partial v_1}{\partial \xi} = 0; \tag{2.1}$$

$$\frac{\partial v_1}{\partial t} + v_0 \frac{\partial v_1}{\partial \xi} + \frac{a_0}{\gamma} \frac{\partial p_1}{\partial \xi} = F_1(\xi)\vartheta[\xi_*(t) - \xi]; \tag{2.2}$$

$$\frac{\partial T_1}{\partial t} + v_0 \frac{\partial T_1}{\partial \xi} + (\gamma - 1) a_0 \frac{\partial v_1}{\partial \xi} = F_2(\xi)\vartheta[\xi_*(t) - \xi], \tag{2.3}$$

where

$$F_1 = \frac{1}{a_0} \frac{\rho_p}{\rho_0} (v_p - v_0) \mu^0(T_0) f(v_p - v_0), \tag{2.4}$$

$$F_2 = \frac{1}{a_0^2} \frac{\rho_p}{\rho_0} [c\lambda (T_p - T_0) g(v_p - v_0) + \gamma(\gamma - 1)(v_p - v_0)^2 f(v_p - v_0)] \mu^0(T_0) \tag{2.5}$$

are known functions depending only on the space variable ξ; $\vartheta(\xi)$ is the theta function.

We consider the compression shock to be fixed at the point $\xi = 0$, as is valid so long as the displacement s of the front is inconsequential relative to the length $l = u_0\tau_0$ of the relaxation zone. Since the stabilization time of the flow behind the shock front is $\sim \tau_u$, in the investigated time range we have $s/l \sim w/u_0 \ll 1$, where w is the perturbation of the wave front velocity. Hereinafter, formally setting $t = \infty$, we interpret this latter time to be of order τ_u, at which time the flow can be regarded as steady.

The boundary conditions at $\xi = 0$ have the form

$$\rho_1 = \alpha v_1, \quad T_1 = \beta v_1, \tag{2.6}$$

where α and β are constants determined from the Rankine–Hugoniot relations. The initial conditions are trivial: $\rho_1 = T_1 = v_1 = 0$.

We now consider the behavior of $F_1(\xi)$ and $F_2(\xi)$. The function $F_1(\xi)$ (2.4) describes the transfer of momentum from the dispersed to the gaseous component. Consequently, $F_1(\xi)$ is everywhere positive and tends to zero for large ξ. It acquires its maximum value immediately behind the shock front, because that is where the difference between the particle and gas-flow velocities is the greatest. The graph of $F_1(\xi)$ is a descending curve (Fig. 1).

The second source $F_2(\xi)$ characterizes the transfer of heat from the gas to the dispersed phase [first term in (2.5)] and the heat release due to the partial dissipation of kinetic energy through friction between the particles and gas flow. For a weak shock wave (curve a) the first term is dominant, so that $F_2(\xi) < 0$ (dashed curve). In the case of a strong shock wave (curve b) the influence of dissipation becomes appreciable, particularly in the immediate vicinity of the shock front. It can be shown that for shock waves of fairly great intensity $F_2(0)$ becomes positive and the graph of $F_2(\xi)$ is represented by the dot–dash curve. The occurrence of a sudden drop of the function $F_2(\xi)$ in the negative domain permits us to divide the relaxation zone somewhat conditionally into two parts, such that heat release due to kinetic energy dissipation is dominant in the first part immediately behind the shock wave and heat transfer from the gas to the particles is dominant in the second part.

We solve the system of equations (2.1)-(2.3) by the method of characteristics. In the postulated setting the characteristic curves are straight lines whose slopes relative to the axis are known and equal to $v_0 + a_0$, $v_0 - a_0$, v_0. Introducing the new dependent variables $S = \gamma T_1 - (\gamma - 1)p_1$, $R_1 = p_1/\gamma + v_1$, $R_2 = p_1/\gamma - v_1$ and replacing the derivatives on the left-hand sides of (2.1)-(2.3) by the derivatives along the characteristic directions, we obtain a system of three ordinary differential equations, the solution of which has the following form in the domain $\xi < a_+ t$:

$$S(\xi, t) = \frac{1}{v_0} \int\limits_{\xi_1(\xi, t)}^{\xi} F_2 d\zeta + S^0\left(t - \frac{\xi}{v_0}\right); \tag{2.7}$$

$$R_1(\xi, t) = \frac{1}{a_+} \int\limits_0^{\xi} \left(F_1 + \frac{F_2}{\gamma}\right) d\zeta + R_1^0\left(t - \frac{\xi}{a_+}\right); \tag{2.8}$$

$$R_2(\xi, t) = -\frac{1}{|a_-|} \int\limits_{\xi}^{\xi_2(\xi, t)} \left(F_1 - \frac{F_2}{\gamma}\right) d\zeta, \tag{2.9}$$

where $a_+ = v_0 + a_0$; $a_- = v_0 - a_0$; S^0, R_1^0 are the values of the functions S and R_1 at the shock discontinuity; and the ordinates $\xi_1(\xi, t)$, $\xi_2(\xi, t)$ of the points of intersection of the contact surface $\xi_*(t)$ with the characteristics of the respective v_0 and a_- families (Fig. 2) through the point (ξ, t) are determined from the equations

$$t - (\xi - \xi_1)/v_0 = t_*(\xi_1), \quad t + (\xi - \xi_2)/|a_-| = t_*(\xi_2),$$

in which $t_*(\xi)$ is the inverse of the function $\xi_*(t)$.

To determine the values of S and R_1 at the compression shock we set $\xi = 0$ in (2.9). The value thus found for $R_2^0(t)$ and conditions (2.6) make it possible to obtain the required S^0 and R_1^0, so that Eqs. (2.7)-(2.9) yield a solution of the stated problem. The physical significance of the terms appearing in the relations obtained for the solution is obvious. Let energy sources be absent, i.e., let $F_2 \equiv 0$. Then, putting $R_2 \equiv 0$, we have

$$v_1 = \rho_1 = \frac{p_1}{\gamma} = \frac{T_1}{\gamma - 1} = \frac{1}{2}\left[\frac{1}{a_+}\int\limits_0^{\xi} F_1 d\zeta + R_1^0\left(t - \frac{\xi}{a_+}\right)\right],$$

whence it is clear that, the integral being positive, the disturbances propagating along the a_+ characteristics are compression waves. Analogously, putting $R_1 \equiv 0$, we arrive at an expression showing that the a_- character-

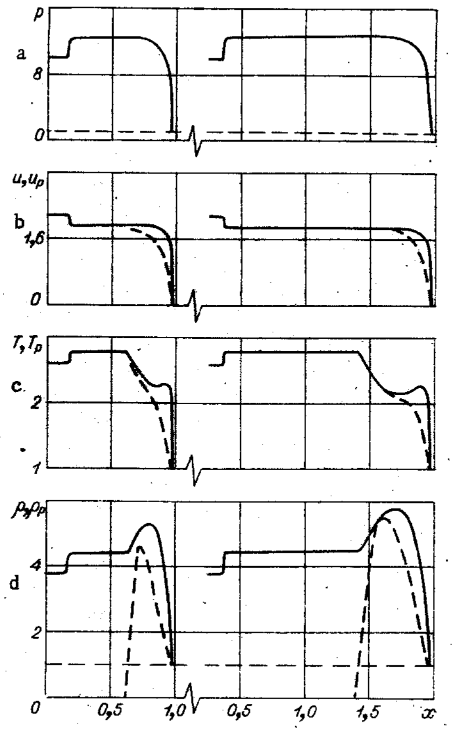

Fig. 3

istics are carriers of rarefaction waves. Both families of waves (compression and rarefaction) are formed by the mutual transfer of momentum between the gaseous and dispersed components. With an energy source present, the amplitude of the resulting waves at a certain point depends on the intensity of the source at that same point. This fact is evinced by the second terms enclosed in parentheses in Eqs. (2.8) and (2.9). Inasmuch as the sources F_1 and F_2 are subsequently distributed continuously throughout the entire domain occupied by the suspended particles, summation of the disturbance takes place along the corresponding characteristics. Moreover, as a result of heat release from the sources, the entropy of a small mass of the gas varies in motion of the latter along the characteristics of the v_0 family (2.7).

§3. Deformation of the Profiles of the Gas Parameter behind the Shock Front. We show that the contact surface $\xi_*(t)$ is a surface of first-order discontinuity of the functions v_1, p_1, and T_1. From relations (2.7)-(2.9) we obtain

$$\left\{\frac{\partial v_1}{\partial \xi}\right\} = \frac{[\dot{\xi}_*(t) - v_0]\, F_1(\xi_*) + \frac{a_0}{\gamma} F_2(\xi_*)}{[a_+ - \dot{\xi}_*(t)]\,[|a_-| + \dot{\xi}_*(t)]}, \qquad \left\{\frac{\partial \rho_1}{\partial \xi}\right\} = \frac{a_0}{\dot{\xi}_*(t) - v_0}\left\{\frac{\partial v_1}{\partial \xi}\right\},$$

$$\left\{\frac{\partial p_1}{\partial \xi}\right\} = \gamma\, \frac{a_0 F_1(\xi_*) + \frac{1}{\gamma}[\dot{\xi}_*(t) - v_0]\, F_2(\xi_*)}{[a_+ - \dot{\xi}_*(t)]\,[|a_-| + \dot{\xi}_*(t)]}, \quad \text{where } \left\{\frac{\partial}{\partial \xi}\right\} = \left.\frac{\partial}{\partial \xi}\right|_{\xi_* - 0} - \left.\frac{\partial}{\partial \xi}\right|_{\xi_* + 0}.$$

It can be shown that, unlike (2.7)-(2.9), the relations for the discontinuities of the derivatives in the time interval from 0 to t_*, where t_* is the root of the equation $\dot{\xi}_*(t) = a_+$, are valid when the contact surface $\xi_*(t)$ is situated above the sonic line $\xi = a_+ t$. This case is realized for a shock wave with supersonic flow of gas behind its front, i.e., when $\dot{\xi}_*(0) > a_+$. For this reason the discontinuities of the derivatives of all parameters of the gas become infinite at the time $t = t_*$ when the velocity of the foremost particle becomes equal to the velocity of the disturbances created by it, corresponding to the formation at the surface $\xi_*(t)$ of a compression shock reflected from the aerocolloid. For shock waves with subsonic flow of the gas behind the front $[\dot{\xi}_*(0) < a_+]$ the effect just described does not happen.

It is evident that the discontinuities of the derivatives of the velocity and pressure at the surface $\xi_*(t)$ tend to zero with time. For the density, on the other hand, we have at large times

$$\{\partial \rho_1/\partial \xi\} \approx (1/\gamma) F_2(\xi_*)/(\dot{\xi}_*(t) - v_0). \tag{3.1}$$

Therefore, depending on the rates at which the numerator and denominator in (3.1) tend to zero, the magnitude of the discontinuity of the derivative of the density, and so also the temperature since $\{\partial p_1/\partial \xi\}_{t \to \infty} 0$, can become equal to zero, a nonzero constant, or infinity. In the last case, which occurs when thermal relaxation is slower than dynamic relaxation, a second-order discontinuity of the density and temperature of the gaseous component is formed at the surface $\xi_*(t)$.

We now consider the variation of the temperature profile of the gas behind the shock front. For simplicity we assume the flow to be steady. It can be shown that for a weak compression shock

$$v_0\frac{dT_1}{d\xi}=\frac{\gamma(\gamma-1)a_0v_0F_1(\xi)+(\gamma-1)\frac{\rho_p}{\rho_0}c\lambda(T_0-T_p)}{a_0^2-v_0^2}>0,$$

i.e., the function $T_1(\xi)$ grows monotonically. In the case of a strong shock (second-order discontinuity)

$$v_0\frac{dT_1}{d\xi}=\frac{\gamma(\gamma-1)a_0v_0F_1(\xi)+\frac{\gamma+1}{2}a_0^2F_2(\xi)}{a_0^2-v_0^2}.$$

Consequently (see Fig. 1), $dT_1/d\xi|_{\xi=0}>0$, i.e., immediately behind the front the temperature increases. At some distance from the shock front, where energy dissipation has become inconsequential, several situations are possible. For example, if the temperature equalization of the particles and gas flow is faster than the velocity equalization, we still have $dT_1/d\xi>0$, so that the temperature profile increases everywhere. But if the temperature relaxation is slower, then the temperature of the gas in this far region is influenced mainly by mutual heat transfer with the particles. In that event

$$v_0\frac{dT_1}{d\xi}\approx-\frac{\rho_p}{\rho_0}\frac{c\lambda(T_0-T_p)}{a_0^2-v_0^2}<0,$$

i.e., the temperature of the gas decreases. Under the indicated conditions, therefore, the temperature in the shock wave exhibits a nonmonotonic variation.

We next determine the nature of the deformations of the profiles of the gas parameters in low-intensity shock waves. We assume that $S^0=R_1^0=0$ at $\xi=0$. This approximation is not too bad for shocks with pressures up to $p_0=2.5$ at the front [6].

For large times we determine the quantities ρ_1, v_1, p_1, and T_1 at the shock front (f), at the gas-aerocolloid contact surface $\xi_*(t)$, and at the contact surface $\xi=v_0t$ representing the trajectory of the gas mass existing at the initial instant at the point $\xi=0$. We have

$$\begin{gathered}\xi=0,\quad \rho_f=-v_f=\frac{p_f}{\gamma}=\frac{T_f}{\gamma-1}=-\frac{1}{2|a_-|}\int_0^\infty\left(F_1-\frac{F_2}{\gamma}\right)d\zeta;\\ \xi=\xi_*(t),\quad \rho_*=v_*=\frac{p_*}{\gamma}=\frac{T_*}{\gamma-1}=\frac{1}{2a_+}\int_0^\infty\left(F_1+\frac{F_2}{\gamma}\right)d\zeta;\\ \xi=v_0t,\quad v_i=\frac{p_1}{\gamma}=v_*=\frac{p_*}{\gamma},\\ \rho_1=\rho_*-\frac{1}{2\gamma v_0}\int_0^\infty F_2d\zeta,\quad T_1=T_*+\frac{1}{2\gamma v_0}\int_0^\infty F_2d\zeta.\end{gathered}\tag{3.2}$$

From the given expressions we deduce the inequalities

$$\rho_f\ p_f,\ T_f,\ T_1<0;\ \rho_*,\ p_*,\ T_*,\ \rho_1>0,$$

and the velocity perturbation is everywhere positive. Moreover, the absolute values of the perturbations of all the parameters of the gaseous component acquire their maximum values at the shock front and their minimum values at the surface $\xi_*(t)$.

§4. Reflected Shock. The existence of a reflected compression shock in the interaction of a shock wave with an aerocolloid is obvious insofar as the compression wave generated thereby, as we showed, propagates in the opposite direction to the incident wave. But the family of compression waves is known to be unstable and with time eventually generates a compression shock.

We now determine the time and point at which the reflected shock is formed. In a coordinate system attached to the gas flow behind the incident shock front with origin at the gas contact surface ($\xi=v_0t$) the given problem is analogous to the problem of the motion of a piston driving into a gas at rest. The difference is that now the piston velocity $\dot{\zeta}(t)=\dot{\xi}_*(t)-v_0$ and the velocity of the gas on its surface $u_\zeta=a_0v_*(t)$ are different. We now confine our discussion to the case of subsonic motion of the contact surface $\zeta(t)$. The coordinate and time dependence of the velocity of the gas ahead of the piston can be written in the implicit form [7]

$$x = \zeta(u) + (a_0 + [(\gamma + 1)/2]u)[t - t_\zeta(u)]. \tag{4.1}$$

From the characteristic intersection condition we obtain the coordinate x_x and time t_x of inception of the shock wave:

$$x_x = \frac{2}{\gamma+1}\frac{a_0[a_0 - \dot{\zeta}(0)]}{\dot{u}_\zeta(0)}, \quad t_x = \frac{2}{\gamma+1}\frac{a_0 - \dot{\zeta}(0)}{\dot{u}_\zeta(0)}. \tag{4.2}$$

Assuming that the reflected shock is of moderate intensity, we can write an expression for the velocity of its front [6]:

$$U = a_0 + [(\gamma + 1)/2]\, u + [(\gamma + 1)^2/32a_0]u^2. \tag{4.3}$$

Differentiating (4.1) and using (4.3), we obtain the equation

$$\frac{(\gamma+1)^2}{32a_0}u\left(u - \frac{8a_0}{\gamma+1}\right)\frac{dt}{du} - \frac{\gamma+1}{2}t = \frac{d}{du}\left[\zeta - \left(a_0 + \frac{\gamma+1}{2}u\right)t_\zeta\right],$$

which has the following solution for the initial condition $t|_{u=0} = t_x$:

$$t = \frac{a_0}{16}\left(1 - \frac{\gamma}{\gamma+1}\frac{a_0}{u}\right)\int_0^u \frac{v\,d\left[\left(a_0 + \frac{\gamma+1}{2}v\right)t_\zeta(v) - \zeta(v)\right]}{\left(a_0 - \frac{\gamma+1}{8}v\right)^3},$$

where the variable of integration v gives in conjunction with (4.1) a parametric definition of the law of motion of the reflected shock front.

An asymptotic ($t \to \infty$) value of the velocity U can be found from (4.3) on the basis of expression (3.2). One particular implication of this operation is that the intensity of the reflected shock increases with the initial concentration of aerocolloidal particles. It also follows from (4.2) that the shock inception time decreases with increasing particle concentration. For transonic flow of the gas behind the incident shock front, however, the numerator in (4.2) becomes small and so the influence of the concentration on the time t_x is less appreciable.

The foregoing results describing the generation and dynamics of the reflected shock have been obtained on the assumption of subsonic flow of the gas behind the incident front. For sonic flow the velocity of the source of disturbances is equal at the initial instant to the propagation velocity of the disturbances themselves. Consequently, a reflected shock is formed instantly. It also follows from (4.2) that $t_x = 0$ in this case.

In the case of supersonic flow behind the shock front, the particles advance ahead of their own disturbances at the outset. Their velocities are equalized at a time determined by the condition of tangency of the sonic line with the contact surface. That time then clearly corresponds to the point of formation of the reflected shock.

§5. Comparison with Numerical Calculations. The analytical solution obtained for the linearized problem of formation of a relaxation shock wave in an aerocolloid has made it possible to advance a number of qualitative considerations about the profiles of the parameters of the gaseous and dispersed components. For a more refined exposition of the fundamental laws we integrate the system (1.1)-(1.7) numerically. We use the Lax-Wendroff difference method [8, 9]. We assume that the undisturbed gas (presumably air) exists at standard conditions.

Typical distributions obtained for the parameters of both components at two times ($t = 0.4$ and $t = 0.84$) by the numerical calculations are given in Fig. 3* relative to the laboratory coordinate system for a shock wave with an initial pressure at the front $p_0 = 10$ and an aerocolloidal cloud of solid particles with $d = 10^{-5}$ m and $c = 1$ at the initial density $\rho_p = 1$. The space variable is measured from the initial position of the gas-aerocolloid boundary, and the time from the instant of passage of the incident shock front across that boundary.

It follows from Fig. 3 that under the influence of the suspended particles on the gas flow the shock wave is deformed. Behind the shock front in this case the parameters of the gas gradually change from their initial values in the incident shock to values corresponding to a stationary relaxation wave. The profiles of the dispersed component undergo an analogous transformation (Fig. 3b-d). Formation of the reflected shock takes place simultaneously.

*Figure 3 does not show the slight oscillations that occur near the shocks and the particle-gas contact surface as finite-difference effects.

Behind the relaxation wave front is observed a relatively extended zone in which momentum and energy are transferred back and forth between the components. The behavior of the profiles of the gas parameters in this zone is qualitatively consistent with the results of the analytical solution. As shown before, the values of the velocity u and pressure p are determined by the superposition of two families of simple compression and rarefaction waves. Compression disturbances are absent at the shock front, and the amplitude of the rarefaction wave has its maximum modulus, so that u and p assume their minimum values there (Figs. 3a and 3b). Compressions begin to play an ever-increasing role with distance from the shock front, and the influence of the rarefaction waves subsides. As a result, u and p grow, acquiring their maximum values at the end of the relaxation zone.

We also note the nonmonotonic behavior of the gas temperature in the relaxation zone (Fig. 3c). The onset of the local T maximum, as already shown, is related to the preponderant influence of energy dissipation over heat transfer behind the shock front (see Fig. 1). The analytically predicted first-order discontinuities of the functions T and ρ at the gas–aerocolloid contact surface also take place (see Fig. 3c and d). But the discontinuities of the derivatives of u and p become significant at earlier times than those given in Fig. 3.

Thus, the results of the analytical solution of the linearized problem and of the numerical solution of the system (1.1)-(1.7) agree completely in the qualitative respect.

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